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SWITCHING CONTROLLER FOR NAVIGATION WITH OBSTACLES IN UNKNOWN ENVIRONMENTS

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Abstract: This paper presents a switching controller for positioning a unicycle-like mobile robot at a desired point with final orientation in completely unknown environments avoiding obstacles. Besides, two algorithms are proposed: the first allows the robot to detect when the obstacle was successfully avoided and the second is intended to decide if the obstacle will be avoided by its right or left side. The obstacle avoidance is performed using a reactive contour-following controller. The switching controllers include the stability analysis at the switching times, using common and multiple Lyapunov functions. Finally, experimental results in a Pioneer III mobile robot show the performance of the proposed controllers.

Keywords: Unicycle mobile robots, multiple Lyapunov functions, switched systems.

1. INTRODUCTION

The problem of programming a mobile robot to move from one place to another is of course as old as the first mobile robot. In mobile robotics almost every task to solve deal with the problem of parking (Aicardi, *et al.*, 1995) or with the classical behavior "move-to-goal" in behavior-based architectures (Arkin, 1998). The unicycle model has a non holonomic constraint that makes it impossible design a continuous invariant control law that guarantees to reach a final posture in Cartesian coordinates. In a seminal paper by Brockett (1983) it is implied that so-called *nonholonomic* systems cannot be stabilized by a differentiable and time-invariant state feedback. Intuitively a nonholonomic constraint restricts a vehicles motion locally but not globally. For the kinematic unicycle, the non holonomic restriction implies no sideways motion of a point on the wheel axis. Note that under this constraint, there is a feasible trajectory between any two configurations (postures). The price paid for free motion of an off-axis point is the lost of orientation control. Several works have been developed in this area; (Fierro, *et al.,* 1996) uses the dynamic model of the mobile robot and achieve the objective by means of neural networks, in (de Wit, *et al.*, 1992; Tayebi, *et al.*, 1997; Aicardi, *et al.,* 1995) a change in the coordinates of the kinematic model have been introduced. Another research objective, related to the robot navigation, is the obstacle avoidance. Regarding this problem many possibilities appear, mainly depending on the kind of obstacle and the inclusion of this behavior into the control architecture (Wang, *et al.*, 2004; Carelli, *et al.*, 2003; Bicho, *et al.*, 2000).

Firstly, this paper presents a switched controller to achieve a desired point with a desired orientation in the Cartesian coordinates. To this aim two controllers were considered, dividing the problem into two tasks: i) orientate the robot and ii) achieve the goal point. The inclusion of the only-heading controller allows to control the geometrical trajectory of the robot, connecting always the initial and final points always by means of a straight line. Then, a contour-following (CF) controller (Toibero, *et al.*, 2006) is considered to avoid almost any unknown obstacle between the initial and the final point. The combination between these strategies allows the robot to handle very real situations, including confinement or trap situations. Significant part of this work is related with the stability of the system. In this context, it is important to mention that: i) the stability of the individual controllers was proved using the Lyapunov theory; ii) the stability at the switching times for the point-to-point controller was proved using a Common Lyapunov Function. Furthermore, iii) stability when reaching the destination point was proved using multiple Lyapunov functions.

The paper is organized as follows. Section 2 gives a brief problem description. Section 3 presents the kinematics of unicycle-like mobile robots. In Section 4 the individual controllers that solve the parking problem are presented, in Section 5 the proposed switched algorithm for obstacle avoidance is described. Then, in Section 6 the supervisor as well as the stability analysis of the overall control system is developed. Finally, Section 7 shows the experimental results.

2. PROBLEM DESCRIPTIONS

This work is divided in three parts: the first is related to the parking problem, i.e. the control of the robot between two arbitrary points. The second part deals with the obstacle avoidance: the chosen algorithm is a CF controller. Then, these two controllers are used together in the third part of this work, which considers the overall problem: the interaction between the robot and the unknown environment around. The robot must reach the final posture $[x_d \quad y_d \quad \theta_d]^T$ starting from any initial posture $[x_i \quad y_i \quad \theta_i]^T$ as can be seen in Fig.1. This problem is depicted with detail in Sections 4, 5 and 6.

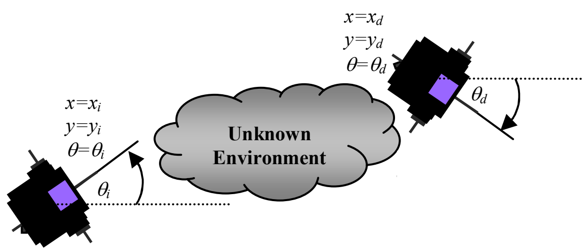

Fig.1. Problem description

3. MOBILE ROBOT

In this paper the wheeled mobile robot of unicycle type shown in Fig.2 is considered, in which the state variables are x, y (the coordinates of the middle point of the rear axle) and θ (angle of the vehicle with the world X-axis [^{W}X]). The rear wheel turns freely and balances the rear end of the robot above the ground. The kinematics of the robot can be modeled by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad (1)$$

where v and ω are the control inputs: the forward and the angular velocity, respectively.

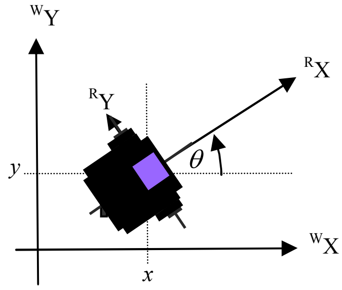

Fig.2. Unicycle mobile robot

The robot is equipped with a laser radar sensor. With reference to Fig.3, the lateral beams from 0º to 15º (and from 165º to 180º) are used to estimate the obstacle contour angle, while all of the beams are used to define a guard-zone (or safety-zone), which purpose is to detect possible robot-obstacle collisions. This rectangular guard-zone is defined by two parameters: the desired lateral (d_{lat}) and frontal (d_{front}) distance as can be shown in Fig.3. The minimum lateral value for a Pioneer IIIDX is about 330 millimeters.

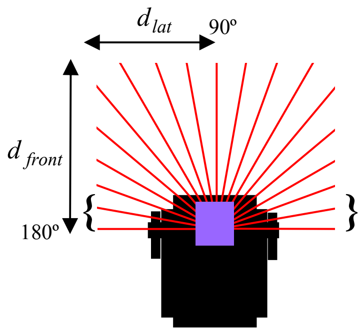

Fig.3. Laser rangefinder

4. PARKING PROBLEM

This section presents a switching controller that solves the parking problem. The two subsystems are continuous controllers described in sections 4.1 and 4.2 respectively. Then, in section 4.3 the switching controller stability is addressed.

4.1 Heading controller

This controller allows positioning the robot at the desired orientation angle θ_d (Fig.4).

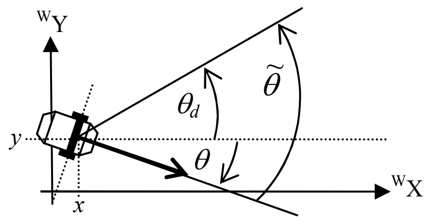

Fig.4. Controller for Angular Position

Considering the angular error

$$\tilde{\theta} = \theta_d - \theta , \tag{2}$$

Its time derivative is

$$\dot{\tilde{\theta}} = -\omega . \tag{3}$$

The following control actions are proposed,

$$v = 0 \tag{4.a}$$

$$\omega = K_{\tilde{\theta}} \tanh(k_\theta \tilde{\theta}), \quad K_{\tilde{\theta}} > 0 \tag{4.b}$$

The expression for the angular velocity saturates at the value of the constant $K_{\tilde{\theta}}$. The value of $k_{\tilde{\theta}} > 0$ is chosen to increase the angular velocity for small errors. Considering the following Lyapunov candidate function,

$$V_{\tilde{\theta}} = \tilde{\theta}^2 / 2 . \tag{5}$$

The asymptotic stability of the system, that is: $\tilde{\theta}(t) \rightarrow 0$, is easily proved.

4.2 Positioning controller

By using this controller (Secci, *et al.*, 1999) the robot can reach a desired point $[x_d \quad y_d \quad \theta]^T$ in the plane, but the heading angle at the target point cannot be adjusted (Fig.5). Considering the errors:

$$\tilde{x} = x_d - x \tag{6.a}$$

$$\tilde{y} = y_d - y . \tag{6.b}$$

The control states are calculated as

$$d = \sqrt{\tilde{x}^2 + \tilde{y}^2} \tag{7.a}$$

$$\tilde{\theta} = \tan^{-1}(\tilde{y} / \tilde{x}) - \theta \tag{7.b}$$

Analyzing the system atthe equilibrium point:

$$\chi = \begin{bmatrix} d \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \tag{8}$$

The time-variation of these states become defined by

$$\dot{d} = -v \cos(\tilde{\theta}) \tag{9.a}$$

$$\dot{\tilde{\theta}} = v \sin(\tilde{\theta}) / d - \omega \tag{9.b}$$

Considering the following Lyapunov candidate function:

$$V_t = \tilde{\theta}^2 / 2 + d^2 / 2 \tag{10}$$

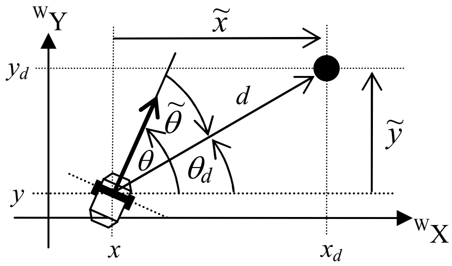

Fig.5. Controller for Target Position

And its derivative along the trajectories

$$\dot{V}_t = \tilde{\theta}\dot{\tilde{\theta}} + d\,\dot{d} = \tilde{\theta}\left(v \sin(\tilde{\theta}) / d - \omega\right) - d\, v \cos(\tilde{\theta}) \tag{11}$$

Defining the control actions

$$v = K_d(d) d \cos(\tilde{\theta}) \tag{12.a}$$

$$\omega = K_d(d) \cos(\tilde{\theta}) \sin(\tilde{\theta}) + K_{\tilde{\theta}} \tanh(k_\theta \tilde{\theta}) \tag{12.b}$$

where the adjustable gain

$$K_d(d) = \frac{v_{max}}{1 + |d|} \geq 0 , \tag{13}$$

It can be concluded the asymptotic stability for this controller at the equilibrium point.

4.3 Switched Controller for parking with final orientation

The block diagram in Fig.6 describes this switching system composed by the controllers described in the previous sections. The switching signal is σ_1. When $\sigma_1 = 1$ the controller for distance correction is activated, while under the cases $\sigma_1 = 0$ or $\sigma_1 = 2$ the controller for Angular Position is activated.

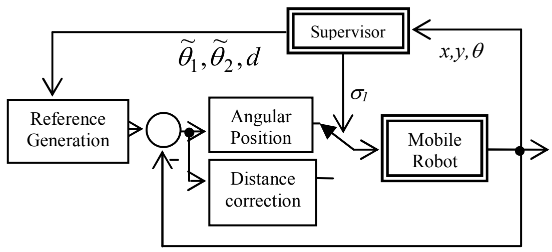

Fig.6. Block diagram of the Supervisor

By redefining the errors according with Fig.7

$$\tilde{\theta}_1 = \theta_{d1} - \theta \tag{14.a}$$

$$\tilde{\theta}_2 = \theta_d - \theta \tag{14.b}$$

$$d = \sqrt{\tilde{x}^2 + \tilde{y}^2} \tag{14.c}$$

and considering as individual subsystems the controllers described in sections 4.1 and 4.2: the switching between these controllers is ruled by an automata, which logics is based on three different stages, a) first the robot is oriented in direction to the desired point by correcting the angle θ_{d1}, b) then, the robot achieves the final point without regard of their orientation and c) finally the robot corrects its orientation to the desired orientation one by making $\theta_d - \theta_{d2} = 0$, as can be seen in Fig.7.

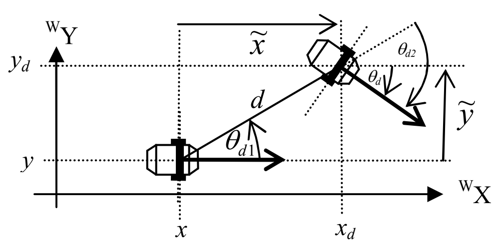

Fig.7. Angles description for the parking controller

A switching-logic graph is depicted in Fig.8, where the initial condition is $\{\sigma_1 = 0, \tilde{\theta} = \tilde{\theta}_1\}$. With the proposed control laws the robot goes straight to the target point using the *heading controller* before and after starting its movement to achieve the target point. In order to demonstrate the stability at the switching times, it is easy to see that (10) is the Common Lyapunov Function for this switched system. Therefore, the supervisor can switch among the mentioned controllers without affecting the stability of the system.

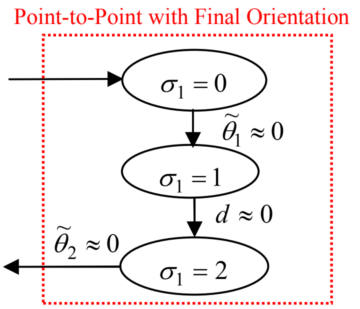

Fig.8. Parking controller: Supervisor logic

In the following simulation results, it is shown a comparison between a common continuous parking controller and the proposed switching controller.

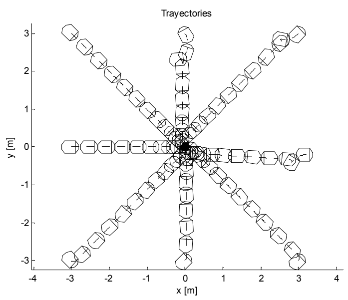

Fig.9. Continuous parking controller without final orientation. Note the path followed by the robot when moving backwards.

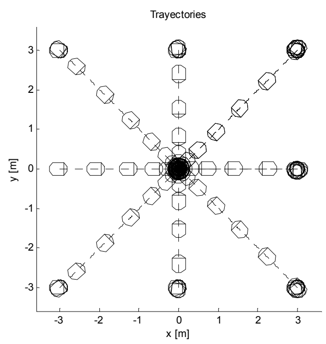

Fig.10. Switching parking controller with final orientation. Always reaching the final point by means of a straight path.

5. SWITCHED CONTROLLER FOR PARKING WITH OBSTACLE AVOIDANCE

Given the robot in its initial position, it must arrive at the desired point without regard of the obstacles between these points (the only requirement is that there should be a feasible path). The proposed solution is not optimal, in the sense that if two or more possible ways are available, the selection of the optimal path it is not performed due to lack of global information. But the way taken by the robot will reach the point in almost all cases, and the overall strategy will be asymptotically stable to the destiny point. When considering obstacles between the initial and final points, it is important to know the moment at which the obstacle has been avoided. To this aim, an appropriate algorithm is proposed.

5.1 Obstacle avoided detection

This algorithm needs to know the actual position of the robot (x,y), the desired final position (x_{REF},y_{REF}) and the position (x_{000},y_{000}) of the laser beam at 0º that indicates the position of the obstacle. These points can be appreciated in Fig.11.

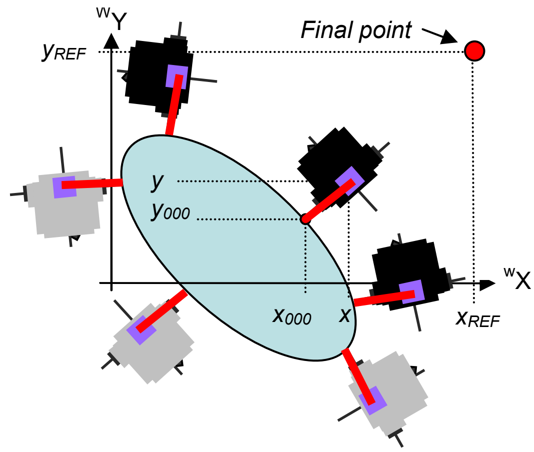

Fig.11. An example of obstacle avoided detection for the third quadrant and for the robot following an obstacle (here with oval shape) at its right side. The black robots indicate the zone where the obstacle was avoided and the grey robots the zones where the obstacle was not yet avoided. Similar graphs can be constructed for the other quadrants and for the robot following the obstacle at its left side

Then the problem is divided into four quadrants depending on the relation between the actual and the final points. A flag variable *OBSTpassed* is defined; the value TRUE for this variable indicates that the obstacle was actually surpassed. As an example for the case in which x_{REF}<x and y_{REF}<x, the algorithm is

$$\begin{aligned}&OBSTACLE_{passed} = false\\&if\{((x_{REF} > x) AND (y_{REF} > y) AND (x_{000} < x) AND (y_{000} < y))\\&OBSTACLE_{passed} = true\}\end{aligned} \quad (15)$$

5.2 Right/Left robot side selection algorithm

As the obstacle avoidance problem is treated by using a CF controller, it is important to detect the side of the robot that will avoid the obstacle. To this aim, the safety-zone defined by the laser range finder is employed, in such a way that, analyzing the obstacle invasion according to Fig.12 (in the next page), it is decided if the robot will avoid the obstacle to its left or to its right side. For brevity the algorithm is not explained in detail, but an intuitive approach can be seen in Fig.12.

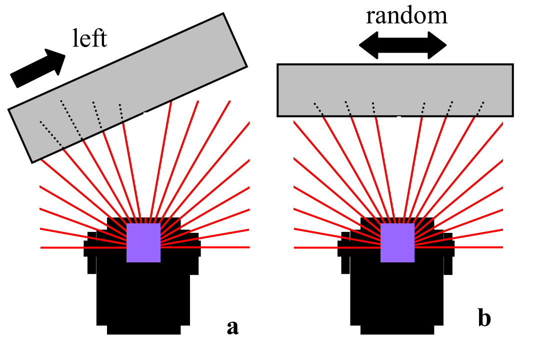

Fig.12. a) Invasion to the safety-zone is stronger at its left side and b) when the invasion is equal at both sides the selection depends on the positions of the goal-point. Finally, if the goal-point is just in front (through the obstacle) then the side to follow is randomly selected.

5.3 Block diagram

The block diagram of the control system takes the form of Fig.13

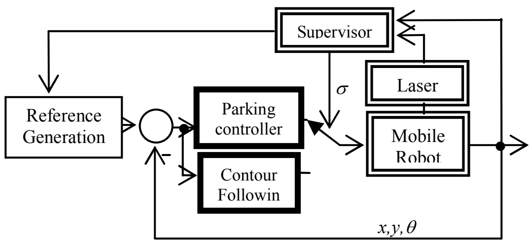

Fig.13. Control system block diagram

So, when σ=0 the robot is approaching the goal point using the parking controller of Section 4.C and only will switch to the CF controller if an obstacle appears. The CF controller allows the robot to follow the discontinuous contour of the obstacle at a desired constant distance.

5.4 Stability Analysis

For the stability analysis of this switching-controller Multiple Lyapunov Functions (Liberzon, 2003) are considerated by associating a Lyapunov function to each controller (one for the parking and other for the CF) and designing a logic that guarantees that the sequence of values for this functions is decreasing. In order to obtain a decreasing sequence for the Lyapunov function that represents the error to the final point (10) the switching logic of Fig.14 is proposed.

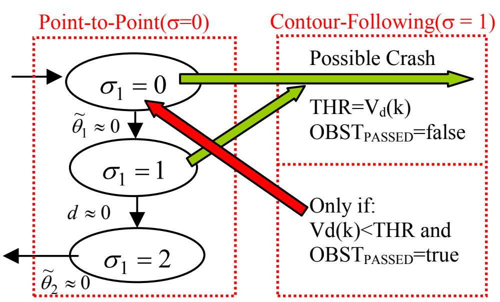

Fig.14. Logic of the supervisor

In practice the logic implements the parking controller until an obstacle is detected, then switch to the CF controller saving the value of Eq. (10) in a threshold variable (THR). The control will return to the parking controller only if the obstacle was avoided and the value of (10) is less than the threshold. Every time the parking controller is activated, it must recalculate the control actions considering the actual point as the initial point, i.e., the robot will head at the desired final point every time that switch to the parking controller.

Note: strictly, the value of the CF controller Lyapunov function must be considered in the same way, i.e., guaranteeing that its sequence be decreasing. But considering that this controller is asymptotically stable, each time that is activated, the associated Lyapunov functions goes quickly to zero while the robot is avoiding the obstacle. For this reason the value of the threshold for this Lyapunov function (Toibero, *et al.*, 2006)

$$V=\frac{1}{2}\tilde{\theta}^2+\int_0^{\tilde{y}} k_{\tilde{y}}(\lambda)\lambda\, d\lambda \qquad (16)$$

can be set as higher as desired on each activation without affecting the performance of the controller.

6. EXPERIMENTAL RESULTS

The experiments were carried out using a Pioneer IIIDX mobile robot. The two first experiments were performed in a laboratory environment in order to show the stability constraints on the logic, while the third was performed along a real office environment.

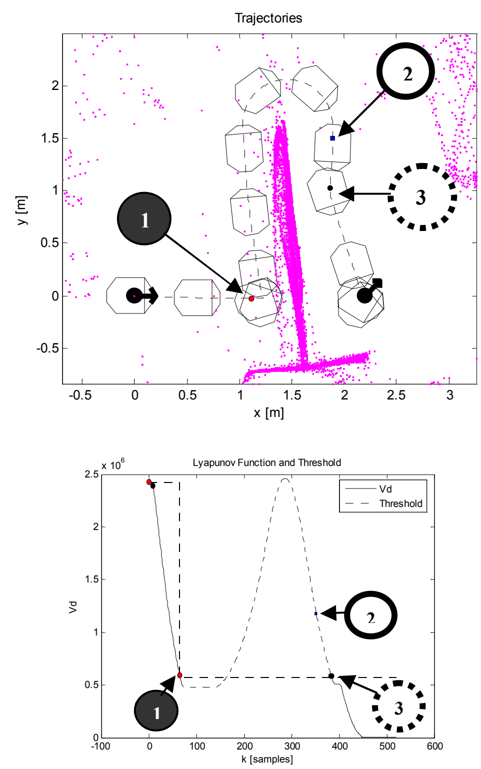

Fig.15. (1): Obstacle detection and the value of Vd is taken as the new threshold value; (2) Obstacle avoided, note that the V_d value is greater than the threshold, so the robot keeps following the obstacle until (3): Value of V_d less than the threshold, switching to the parking controller. Note that V_d is decreasing due to the selected logic.

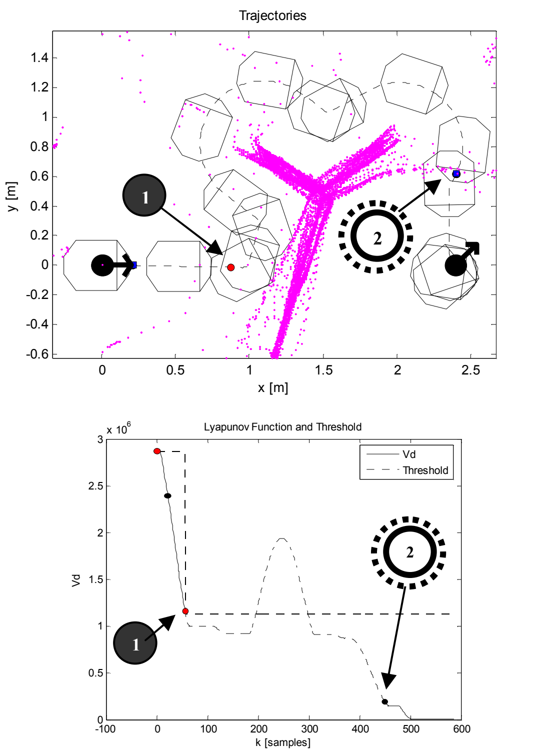

Fig.16. (1): obstacle detected; (2): obstacle avoided with a Vd value minor than the threshold. This is a direct switching case.

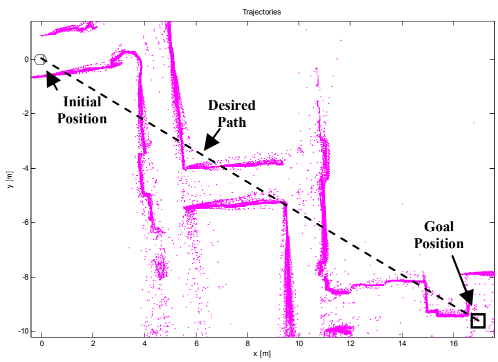

Fig.17. Large-scale experiment setting (20meters long),

CONCLUSIONS

In this paper it has been presented a switching controller that deals with the problem of positioning a mobile robot with final orientation by avoiding unknown obstacles. Besides, two algorithms were proposed: one that allows the robot to detect when an obstacle was or not avoided; and another that selects the side to avoid the obstacle. The presented switching controllers have included the stability analysis at the switching instants. Finally, experimental results have shown the performance of the controller in real situations.

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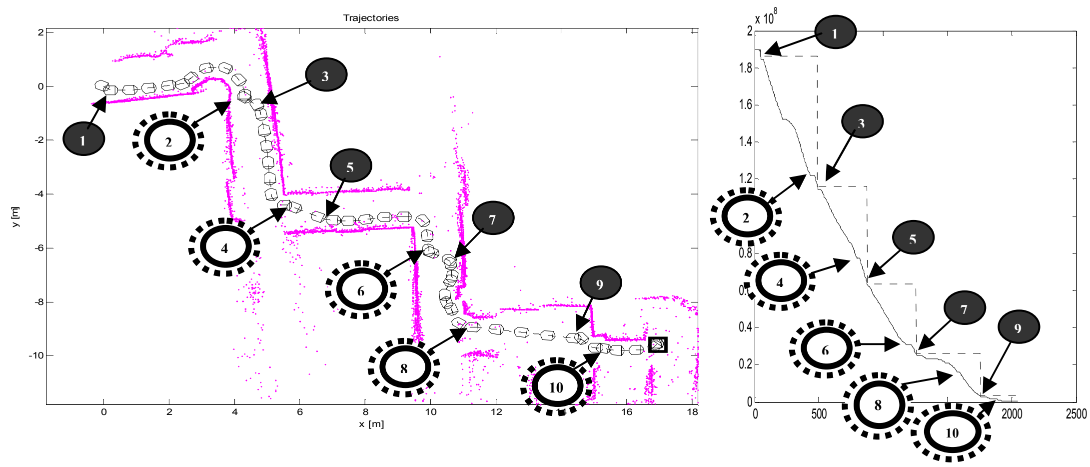

Fig.18. Left picture: (1,3,5,7,9) obstacle detection; (2,4,6,8,10): obstacle avoided. Right Picture: Lyapunov function.