AN OBSERVER-BASED FEEDFORWARD/FEEDBACK CONTROLLER FOR ROBOTIC MANIPULATORS

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Abstract: This paper presents a nonlinear control strategy for robotic manipulators. The controller combines a nonlinear feedback law with a feedforward one. In order to reduce the number of sensors a nonlinear observer is used to estimate both states and disturbances from measured variables. In this way, an observer-based feedback/feedforward controller is designed. The proposed controller is applied to a two-link manipulator for illustrating its performance.

Keywords: robotic manipulator, observer-based control, feedforward

1. INTRODUCTION

Robotic manipulators performance can be improved by rejecting disturbances via a feedforward strategy. In many cases, a feedforward controller is combined with a feedback controller for obtaining good performance of the controlled system.

When feedforward compensation is used, sensors are needed for measuring disturbances. In industry applications this appears as a drawback to be overcome; thus, designers employ different techniques for avoiding those sensors. A widely used technique incorporates good estimates of the disturbances. Since an estimate is not the actual value of the disturbance, system performance is slightly deteriorated when the actual disturbances values are replaced by their estimates. However, there are two main reasons for using an observer-based controller: (a) deterioration could be insignificant if good estimates are included in the feedforward controller and (b) when estimates are constructed from other measured variables many sensors could be eliminated.

A well-known procedure of estimating disturbances is to use disturbance observers (DO). These observers have been employed in both linear and nonlinear control strategies in many applications (see (Chen *et al.*, 2004)(Chen *et al.*, 2000*b*)(Yang *et al.*, 2005) among others and references in these papers). As above mentioned, estimates are used for designing a feedforward compensator and this compensator can be combined with a feedback strategy for constructing a feedforward/feedback controller.

Based upon this idea, recently in (Chen *et al.*, 2000*a*) and (Chen, 2004) a disturbance observer-based controller has been introduced. In (Chen *et al.*, 2000*a*) a nonlinear disturbance observer is employed for compensating the friction for a two-link robotic manipulator. In (Chen, 2004) a general

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framework is presented and the controller is tested over a two-link robotic manipulator. Although the performance is improved by including a DO for estimating disturbances (or unknown dynamics), in the controller proposed in (Chen, 2004), all the state variables need to be measured.

In spite of the fact that the DO-based controller is useful in some cases, a more general scheme should consider a nonlinear observer (NO) estimating disturbances and states from other measured variables. The main advantage of this scheme consists in reducing the number of sensors.

The main goal in this paper is to present an output control strategy based on a nonlinear observer. The performance of this controller is almost the same as that of the DO-based controller introduced in (Chen, 2004). By using the proposed scheme many sensors could be avoided depending on the case under consideration. By taking into account the whole system, it can be mentioned that during the brief time needed to the estimated states convergence to their the actual values the NO observer-based controller performance slightly differs from that of the DO observer-based controller. Nevertheless, in practice the advantages of reducing the number of sensors are many (for instance, a scheme with a reduced number of sensors is less expensive, rugger and easier to maintain).

In this paper, the proposed controller is applied for tracking velocities in a two-link manipulator robot. In this case, only position sensors are used for estimating both velocities and disturbances. The closed-loop scheme performs very well. It is important to remark that the controller proposed in (Chen *et al.*, 2000*a*) and (Chen, 2004) not only requires position sensors but velocity sensors as well. On the contrary, by using our proposal a high-performance controller is obtained and in comparison with the controller introduced in (Chen, 2004), velocity sensors are not needed. The paper is organized as follows. In section 2, the controller is introduced. In section 3, the control strategy is applied to a two-link robotic manipulator. Finally, conclusions are drawn in section 4.

2. NONLINEAR OBSERVER BASED CONTROL

Consider a nonlinear system given by (see (Chen, 2004))

$$\dot{\chi} = f(\chi) + g(\chi)u + g_d(\chi)d \tag{1}$$

$$y = h(\chi) \tag{2}$$

where $\chi \in R^n$, $u \in R^v$, $y \in R^p$ and $d \in R^d$ are the state vector, input, output and external disturbance, respectively. In addition, consider disturbances generated by a nonlinear exosystem assumed to be neutrally stable and given by

$$\dot{\xi} = f_\xi(\xi) \tag{3}$$

$$d = h_d(\xi) \tag{4}$$

where $\xi \in R^s$. Then, an extended system can be attained by combining (1)-(4), such that

$$\dot{\chi} = f(\chi) + g(\chi)u + g_d(\chi)h_d(\xi) \tag{5}$$

$$\dot{\xi} = f_\xi(\xi) \tag{6}$$

$$y = h(\chi) \tag{7}$$

Taking into account the system given by (5)-(7), it must be remarked that in many cases both vectors (i.e. χ and ξ) can be estimated from the measured variables (y). When DO is used, only the vector ξ is estimated. Moreover, note that either a reduced or a full order observer can be constructed for the system (5)-(7) (see (Solsona *et al.*, 2000) and (DeAngelo *et al.*, 2003) for examples).

2.1 A nonlinear observer

It is well known that there exist several techniques for designing nonlinear observers. Among others, it is possible to mention the nonlinear Luenberger-like observer, variable structure observer, extended Kalman filter, passivity based observer and linear dynamics observer. However, analysis of different techniques is beyond the scope of this paper.

In what follows, a nonlinear Luenberger-like observer is designed (Cicarrella *et al.*, 1993)(Dalla-Mora *et al.*, 2000) for constructing an $\eta = [\chi^T \quad \xi^T]^T$ estimator, where $\eta \in R^m$ and $m = n + s$. Note that η contains both states and disturbances. By rewritting (5)-(7), we obtain

$$\dot{\eta} = \varphi(\eta) + \phi(\eta)u \tag{8}$$

$$y = \psi(\eta) \tag{9}$$

with $\varphi(\eta) = f(\chi) + g_d(\chi)h_d(\xi)$, $\psi(\eta) = h(\chi)$ and $\phi(\eta) = g(\chi)$.

Then, given the system represented by (8) and (9), an observer for estimating η can be constructed measuring y. A procedure for constructing the observer is developed below. In order to obtain the observer, the following nonlinear transformation is used,

$$z = [z_1 \dots z_m]^T = [\gamma_1(\eta) \dots \gamma_m(\eta)]^T \in R^m \tag{10}$$

where z is the state vector in new coordinates and it is assumed that $m > p$ (see $\gamma_i(\eta)$ definition in eqns. (11)-(23)). Therefore, a transformed representation becomes

$$
\begin{aligned}
y_1 &= \psi_1(\eta) = \gamma_1(\eta) = z_1 && (11)\\
\dot{z}_1 &= \frac{\partial \gamma_1}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_2(\eta) + \mu_2(\eta) u = \\
&= z_2 + \mu_2(\eta) u && (12)\\
\dot{z}_2 &= \frac{\partial \gamma_2}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_3(\eta) + \mu_3(\eta) u = \\
&= z_3 + \mu_3(\eta) u && (13)\\
&\vdots \\
\dot{z}_{l_1-1} &= \frac{\partial \gamma_{l_1-1}}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_{l_1}(\eta) + \mu_3(\eta) u = \\
&= z_{l_1} + \mu_{l_1}(\eta) u && (14)\\
\dot{z}_{l_1} &= \frac{\partial \gamma_{l_1}}{\partial \eta}(\varphi + \phi u) = \sigma_1 + \delta_1 u && (15)
\end{aligned}
$$

$$
\begin{aligned}
y_2 &= \psi_2(\eta) = \gamma_{l_1+1}(\eta) = z_{l_1+1} && (16)\\
\dot{z}_{l_1+1} &= \frac{\partial \gamma_{l_1+1}}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_{l_1+2}(\eta) + \mu_{l_1+2}(\eta) u = \\
&= z_{l_1+2} + \mu_{l_1+2}(\eta) u && (17)\\
&\vdots \\
\dot{z}_{l_1+l_2-1} &= \frac{\partial \gamma_{l_1+l_2-1}}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_{l_1+l_2}(\eta) + \mu_{l_1+2}(\eta) u = \\
&= z_{l_1+l_2} + \mu_{l_1+l_2}(\eta) u && (18)\\
\dot{z}_{l_1+l_2} &= \frac{\partial \gamma_{l_1+l_2}}{\partial \eta}(\varphi + \phi u) = \\
&= \sigma_2(\eta) + \delta_2 u && (19)
\end{aligned}
$$

$$
\begin{aligned}
&\vdots \\
y_p &= \psi_p(\eta) = \gamma_{l_1+\cdots+l_{p-1}+1}(\eta) = \\
&= z_{l_1+\cdots+l_{p-1}+1} && (20)\\
\dot{z}_{l_1+\cdots+l_{p-1}+1} &= \frac{\partial \gamma_{l_1+\cdots+l_{p-1}+1}}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_{l_1+\cdots+l_{p-1}+2}(\eta) + \\
&\mu_{l_1+\cdots+l_{p-1}+2}(\eta) u = \\
&= z_{l_1+\cdots+l_{p-1}+2} + \\
&= \mu_{l_1+\cdots+l_{p-1}+2}(\eta) u && (21)\\
&\vdots \\
\dot{z}_{l_1+\cdots+l_p-1} &= \frac{\partial \gamma_{l_1+\cdots+l_p-1}}{\partial \eta}(\varphi + \phi u) = \\
&= \gamma_{l_1+\cdots+l_p} + \mu_{l_1+\cdots+l_p}(\eta) u = \\
&= z_{l_1+\cdots+l_p} + \\
&= \mu_{l_1+\cdots+l_p}(\eta) u && (22)
\end{aligned}
$$

$$
\begin{aligned}
\dot{z}_{l_1+\cdots+l_p} &= \frac{\partial \gamma_{l_1+\cdots+l_p}}{\partial \eta}(\varphi + \phi u) = \\
&= \sigma_p(\eta) + \delta_p u && (23)
\end{aligned}
$$

where $\sum_{k=1}^{p} l_k = m$. Assuming that $\frac{\partial \gamma}{\partial \eta}$ is non-singular in η, there exists; $\eta = \tilde{\gamma}(z)$, which means that the vector η (i.e., in original coordinates) can be calculated from the knowledge of the vector z via $\tilde{\gamma}$ (by using the Inverse Function Theorem). In new coordinates, the model given by (8)-(9) results

$$
\begin{aligned}
\dot{z} &= A\, z + \rho(z) + \pi(z) u && (24)\\
y &= C\, z && (25)
\end{aligned}
$$

where

$$\rho(z) = [0 \cdots \sigma_1(\eta) \quad 0 \cdots \sigma_2(\eta) \cdots \quad \sigma_p(\eta)]^T|_{(\eta=\tilde{\gamma}(z))}$$

and

$$\pi(z) = [\mu_1 \cdots \delta_1(\eta) \quad \mu_2 \cdots \delta_2(\eta) \cdots \quad \delta_p(\eta)]^T|_{(\eta=\tilde{\gamma}(z))}$$

From the comparison of (24) and (25) with equations (11)-(23), the following assignment is obtained:

$$
A = \begin{bmatrix} A_1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & A_k & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & A_p \end{bmatrix} \tag{26}
$$

where A_k is a matrix of dimension $l_k \times l_k$, with $k = 1, \cdots, p$, given by

$$
A_k = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{27}
$$

and C is a matrix of dimension $p \times m$ given by

$$
C = \begin{bmatrix} C_1^1 \\ C_2^{l_1+1} \\ \vdots \\ C_i^j \\ \vdots \\ C_p^{l_1+\cdots+l_{p-1}+1} \end{bmatrix} \tag{28}
$$

where C_i^j, with $j = 1, \ldots, \sum_{k=1}^{(j-1)} l_k + 1$, is a row vector with 1 in the position j and zero otherwise. Under this construction, the pair (C,A) is observable. Considering the nonlinear system given by (24) and (25) and assuming that $\rho(z) + \pi(z)u$ is Lipschitz in z, uniformly in u, with a

Lipschitz constant L, an asymptotic estimator from z is given by

$$\dot{\hat{z}} = A\hat{z} + \rho(\hat{z}) + \pi(\hat{z})u + G\left(y - C\hat{z}\right) \qquad (29)$$

with G a constant matrix, when there exist P and Q positive definite matrices satisfying

$$(A - GC)^T P + P(A - GC) = -Q \qquad (30)$$

and

$$-\lambda_Q^{min} + 2\lambda_P^{max} L < 0 \qquad (31)$$

where λ_Q^{min} is the minimum eigenvalue of matrix Q, λ_P^{max} is the maximum eigenvalue of matrix P (see appendix B for a demonstration).

After $\hat{z}$ has been calculated, the estimated variables in original coordinates ($\hat{\eta}$) are obtained using the function $\tilde{\gamma}$,

$$\hat{\eta} = \tilde{\gamma}(\hat{z}) \qquad (32)$$

Note that $\hat{\eta}$ contains states and disturbances estimates.

2.2 The Controller

Several nonlinear control laws can be used for controlling robotic manipulators. For instance, feedback linearization (Isidori, 1995), IDA-PBC (Ortega and García-Canseco, 2004) (interconnection and damping assigment passivity-based control) or back-stepping (Dawson *et al.*, 1998). High performance schemes are attained by using these control laws. Nevertheless, in many applications states measurements are needed for implementing the above mentioned control laws. In order to reduce the numbers of sensors, observer-based controllers can be designed. In these cases, controller parameters can be chosen for guaranteeing the whole system stability. It can be done by using several techniques. For instance, in (Chen, 2004) Lyapunov theory is used. Others techniques can be found in (Etchechoury *et al.*, 2001) and references there in.

3. AN EXAMPLE

Consider the DO-based control applied to a two-link robotic manipulator. In this case, DO-based control law needs position and velocity sensors to be implemented (see (Chen *et al.*, 2000*a*) and (Chen, 2004)). However, our proposal can be used for avoiding velocity sensors. In a previous work (Solsona and Puleston, 2000) a reduced-order nonlinear observer was designed for estimating velocities in N-link manipulators. Nevertheless, disturbances were not taken into account in that work. In spite of that it is possible to extend the system by including the disturbance dynamics and by designing an observer for estimating velocities and disturbances from the positions measurements. Consider the model of the rigid two-link manipulator resulting from Lagrange equations (Solsona and Puleston, 2000)

$$J(q)\ddot{q} = \tau - C(q, \dot{q})\dot{q} - \tau_g(q) \qquad (33)$$

where q, $\dot{q}$, $\ddot{q}$, are positions, velocities, and accelerations, respectively. τ is the control torque, $J(q)$ is the definite positive inertia matrix, $C(q, \dot{q})$ is the Coriolis and centripetal matrix, and τ_g are the gravity components. Then, from (33) and including distubances (ξ_1,ξ_2), a state-space model results in

$$\dot{x}_1 = x_3 \qquad (34)$$

$$\dot{x}_2 = x_4 \qquad (35)$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = J^{-1}(x_1, x_2)\Bigg(\tau - C(x)\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} - \tau_g(x_1, x_2)\Bigg) + \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \qquad (36)$$

$$\dot{\xi}_1 = 0 \qquad (37)$$

$$\dot{\xi}_2 = 0 \qquad (38)$$

with $x = [x_1, x_2, x_3, x_4]^T$. In this case, a nonlinear reduced-order observer for estimating velocities and disturbances from positions measurements can be designed as follows (see (Solsona and Puleston, 2000) for the design of a reduced-order observer estimating velocities from positions measurements).

$$\begin{bmatrix} \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{\hat{\xi}}_1 \\ \dot{\hat{\xi}}_2 \end{bmatrix} = \begin{bmatrix} J^{-1}(x_1, x_2)\Bigg(\tau - C(\hat{x})\begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \\ -\tau_g(x_1, x_2)\Bigg) + \begin{bmatrix} \hat{\xi}_1 \\ \hat{\xi}_2 \end{bmatrix} \\ 0 \\ 0 \end{bmatrix} + G\begin{bmatrix} \dot{x}_1 - \hat{x}_3 \\ \dot{x}_2 - \hat{x}_4 \end{bmatrix}, \qquad (39)$$

with $\hat{x} = [x_1, x_2, \hat{x}_3, \hat{x}_4]^T$. In order to avoid differentiating the measured positions, a simple change of variables can be used, so that the equations to be implemented become

$$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \end{bmatrix} = \begin{bmatrix} J^{-1}(x_1, x_2)\Bigg(\tau - C(\hat{x})\begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \\ -\tau_g(x_1, x_2)\Bigg) + \begin{bmatrix} \hat{\xi}_1 \\ \hat{\xi}_2 \end{bmatrix} \\ 0 \\ 0 \end{bmatrix} - G\begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \qquad (40)$$

$$\begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \\ \hat{\xi}_1 \\ \hat{\xi}_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} + G\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \qquad (41)$$

By using the estimated variables, nonlinear control laws can be constructed as proposed in (Chen, 2004). For instance, by using either feedback linearization technique (Isidori, 1995) or IDA-PBC method (Ortega and García-Canseco, 2004) the following law for speed tracking is obtained,

$$\begin{aligned}\tau = J(x_1, x_2)\left\{\begin{bmatrix}\dot{x}_{3_{ref}}\\ \dot{x}_{4_{ref}}\end{bmatrix} - \begin{bmatrix}\hat{\xi}_1\\ \hat{\xi}_2\end{bmatrix} -\right.\\ \left. - K_C\begin{bmatrix}(\hat{x}_3 - x_{3_{ref}})\\ (\hat{x}_4 - x_{4_{ref}})\end{bmatrix}\right\} +\\ +C(\hat{x})\begin{bmatrix}\hat{x}_3\\ \hat{x}_4\end{bmatrix} + \tau_g(x_1, x_2), \qquad (42)\end{aligned}$$

with K_C a constant matrix gain.

Note that the control law (42) uses positions measurements, estimated velocities and estimated disturbances. In this way, when this control strategy is used velocity sensors are avoided. Taking into account a two-link robotic manipulator whose data and parameters are given in appendix A, speed-tracking performance is illustrated in Figures 1-4. Whereas Fig. 1 and 2 show velocities references and velocities in links 1 and 2, respectively; velocities tracking errors are drawn in Figs. 3 and 4.

4. CONCLUSIONS

In this paper a nonlinear observer-based controller for robotic manipulators has been proposed. In order to obtain good performance, the controller combines feedback and feedforward strategies. Feedforward law is used for rejecting disturbances. However, disturbance sensors are avoided by employing a nonlinear observer. The observer estimates both states and disturbances such that the proposed strategy results less expensive than others found in the literature. As an example, the control strategy is tested on a two-link robotic manipulator.

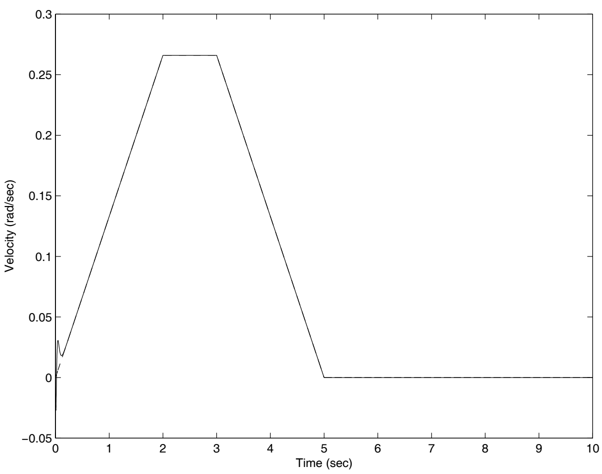

Fig. 1. Link 1, velocity reference and actual velocity.

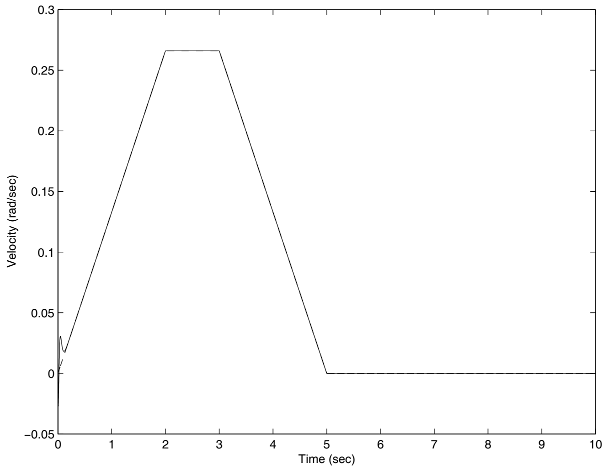

Fig. 2. Link 2, velocity reference and actual velocity.

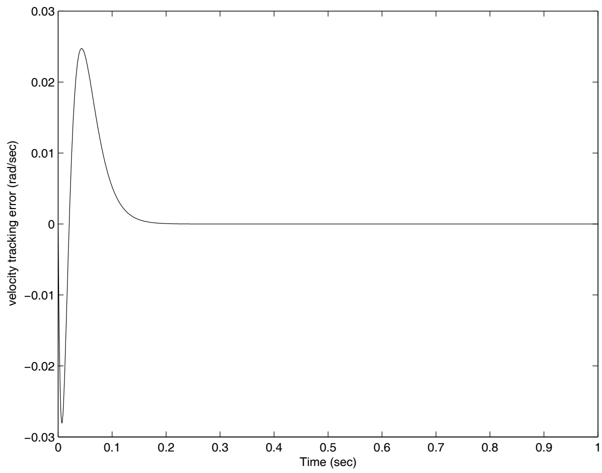

Fig. 3. Link 1, velocity tracking error

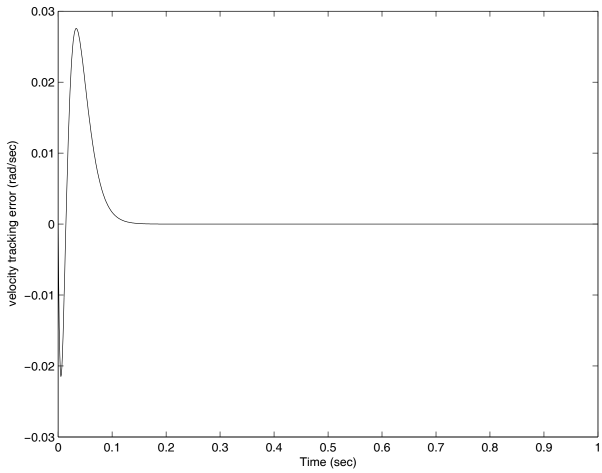

Fig. 4. Link 2, velocity tracking error

5. ACKNOWLEDGMENTS

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APPENDIX A

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$J_{11} = m_1 lc_1^2 + m_2(l_1^2 + lc_2^2 + 2l_1 lc_2 cos(x_2)) + I_1 + I_2$$
$$J_{12} = m_2(lc_2^2 + l_1 lc_2 cos(x_2)) + I_2$$
$$J_{21} = m_2(lc_2^2 + l_1 lc_2 cos(x_2)) + I_2$$
$$J_{22} = m_2 lc_2^2 + I_2$$

$$C = \begin{bmatrix} -m_2 l_1 lc_2 sin(x_2) x_4 & -m_2 l_1 lc_2 sin(x_2)(x_3 + x_4) \\ -m_2 l_1 lc_2 sin(x_2) x_3 & 0 \end{bmatrix}$$

$$\tau_g = \begin{bmatrix} (m_1 lc_1 + m_2 l_1) g sin(x_1) + m_2 lc_2 g sin(x_1 + x_2) \\ m_2 lc_2 g sin(x_1 + x_2) \end{bmatrix}$$

Manipulator parameters: $l_1 = 0.45\,m, l_2 = 0.55\,m, lc_1 = 0.091\,m, lc_2 = 0.105\,m, m_1 = 23.90\,kg, m_2 = 4.44\,kg, I_1 = 1.27\,kgm^2, I_2 = 0.24\,kgm^2, g = 9.8\,m/s^2$.

Controller parameters:

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \\ g_{41} & g_{42} \end{bmatrix} \quad K_C = \begin{bmatrix} k_{1_c} & 0 \\ 0 & k_{2_c} \end{bmatrix}$$

$k_{1_c} = 100, k_{2_c} = 100, g_{11} = 110, g_{12} = 0, g_{21} = 0, g_{22} = 150, g_{31} = 3000, g_{32} = 0, g_{41} = 0, g_{42} = 5600$.

Initial conditions: $x_1 = 0.314\,rad, x_2 = 0.4\,rad, \hat{x}_3 = 0.1\,rad/s, \hat{x}_4 = 0.1\,rad/s$,
$\xi_1 = 0.013\,rad/s^2, \xi_2 = 0.013\,rad/s^2, \hat{\xi}_1 = 0.01\,rad/s^2, \hat{\xi}_2 = 0.01\,rad/s^2$.

APPENDIX B

Let $e_z = z - \hat{z}$ be the estimation error. Then, by substracting (29) from (24) the estimation error dynamics results in

$$\frac{de_z}{dt} = (A - GC)e_z + \rho(z) - \rho(\hat{z}) + (\pi(z) - \pi(\hat{z}))u$$

Consider the Lyapunov candidate function $V = e_z^T P e_z$, then

$$\frac{dV}{dt} = \frac{de_z^T}{dt} P e_z + e_z^T P \frac{de_z}{dt}. \tag{43}$$

Since $\rho(z) + \pi(z)u$ is assumed to be Lipschitz in z, uniformly in u, with Lipschitz constant equal to L, there exists L such that $\|\rho(z) - \rho(\hat{z}) + (\pi(z) - \pi(\hat{z}))u\| \le L\|e_z\|$, consequently (43) can be bounded as follows

$$\frac{dV}{dt} \le \left(-\lambda_Q^{min} + 2\lambda_P^{max} L\right) \|e_z\|^2.$$

When $-\lambda_Q^{min} + 2\lambda_P^{max} L < 0$ is satisfied, the derivative of the Lyapunov candidate function is negative such that the estimation error converges to zero.

REFERENCES

Chen, W. (2004). Disturbance observer based control for nonlinear systems. *IEEE/ASME Trans. on Mechatronics* **9**, 706–710.

Chen, W., D. Ballance, P. Gawthrop and J. O'Reilly (2000*a*). A nonlinear disturbance observer for robotic manipulators. *IEEE Trans. on Industrial Electronics* **47**, 932–938.

Chen, X., C. Su and T. Fukuda (2004). A nonlinear disturbance observer for multivariable system and its application to magnetic bearing systems. *IEEE Trans. on control systems technology* **12**, 569–577.

Chen, X., S. Kornada and T. Fukuda (2000*b*). Design of the nonlinear disturbance observer. *IEEE Trans. on Industrial Electronics* **47**, 429–437.

Cicarrella, G., M. DallaMora and A. Germani (1993). A luenberger-like observer for nonlinear systems. *Int. Journal of Control* **57**, 537–556.

DallaMora, M., A. Germani and C. Manes (2000). Design of state observers from a drift-observability property. *IEEE Trans. on Automatic Control* **45**, 1536–1540.

Dawson, D., J. Hu and T. Burg (1998). *Nonlinear control of electric machinery.* Marcel Dekker, Inc.

DeAngelo, C., G. Bossio, J. Solsona, G. García and M. I. Valla (2003). Sensorless speed control of permanent magnet motors driving an unkwown load. *CD Proceedings of the 2003 IEEE International Symposium on Industrial Electronics (ISIE'03).*

Etchechoury, M., J. Solsona and C. Muravchik (2001). Feedback linearization via state transformation using estimated states. *International Journal of Systems Science* **32**, 1–7.

Isidori, A. (1995). *Nonlinear control systems.* Springer-Verlag.

Ortega, R. and E. García-Canseco (2004). Interconnection and damping assignment passivity-based control: a survey. *European Journal of Control* **10**, 432–450.

Solsona, J. and P. Puleston (2000). A reduced order observer for velocity estimation of n-link manipulators. *Int. Journal of Robotics and Automation* **15**, 51–56.

Solsona, J., M.I. Valla and C. Muravchik (2000). Nonlinear control of a permanent magnet synchronous motor with disturbance torque estimation. *IEEE Trans. on Energy Conversion* **15**, 163–168.

Yang, K., Y. Choi and W.K. Chung (2005). On the tracking improvement performance of optical disk drive servo system using error-based disturbance observer. *IEEE Trans. on Industrial Electronics* **52**, 270–279.